Final review!

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- Computability (constructing e.g.'s)
- Joint dist, given diagram
- Normal dist, exp. cutoff ← 0t3c cond.
- Cond. exp.
- Union bound
- Geom, exp.
- E, Var of cond. in discrete

Computability

"This statement is false"

If true ⟹ false

If false ⟹ true

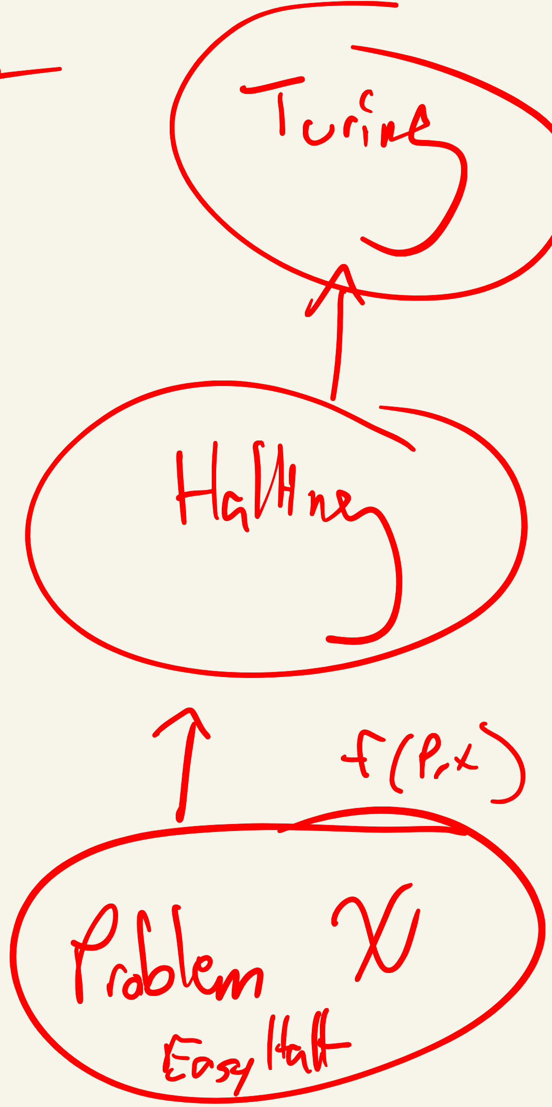
Halting(P, x):
    If (program P halts on input x)
        return 1
    else
        return 0

"Turing(Turing)"?

If Turing halts ⟹ loops
loops ⟹ halts

Turing(P)
    if Halting(P, P):
        # loop forever
    else
        # halt

Computability



(basically "this statement is false")

```
def f(P,x)
    ...        <- Problem X
    ...
    ...
    return y
```

f(P,x) = Halting(P,x) for all P,x

Comp

① IF P halts on X, f(P,X) = 1 ✓

② IF P loops on X, f(P,X) = 0 ✓

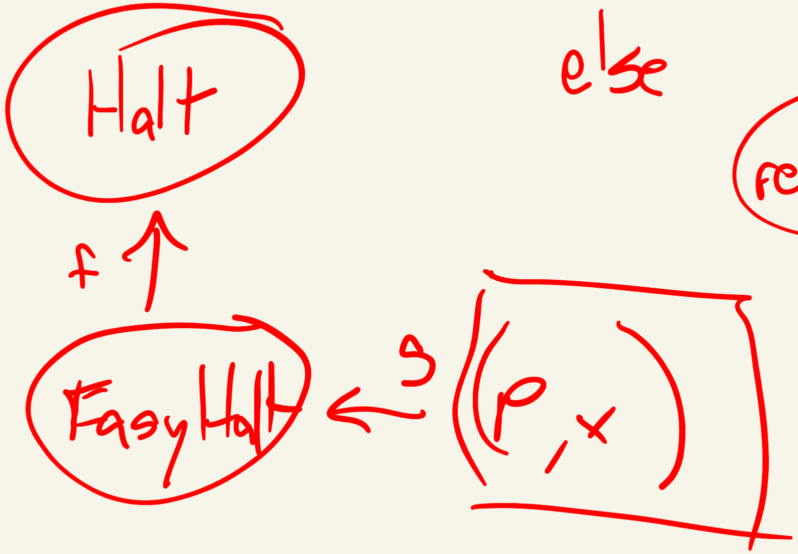
e.g. → EasyHalt(P)

if (P halts on input 0)

return 1

else

return 0




```
def f(x, P):
    def g(y):
        return P(x)
    return EasyHalt(g)
```

0:35

$\mathbb{P} : \mathcal{P}(\Omega) \to [0,1]$

$\{0,1,2,3\}$

$\mathbb{P}(\{0\}), \mathbb{P}(\{1\}), \dots$

$$\mathbb{P}(E) = \sum_{x \in E} \mathbb{P}(\{x\})$$

$$=: \sum_{x \in E} p(x)$$

$\mathbb{P} : \mathcal{P}(\Omega) \to [0,1]$

$\Omega \subset \mathbb{R}$

$\mathbb{P}(X \le k)$

CDF

$CDF(k) \quad k \to \mathbb{P}(X \le k)$

$$\mathbb{P}(a \le X \le b) = \mathbb{P}(X \le b) - \mathbb{P}(X \le a)$$

$$= \int_a^b CDF'(x)\,dx$$

$$=: \int_a^b p(x)\,dx$$

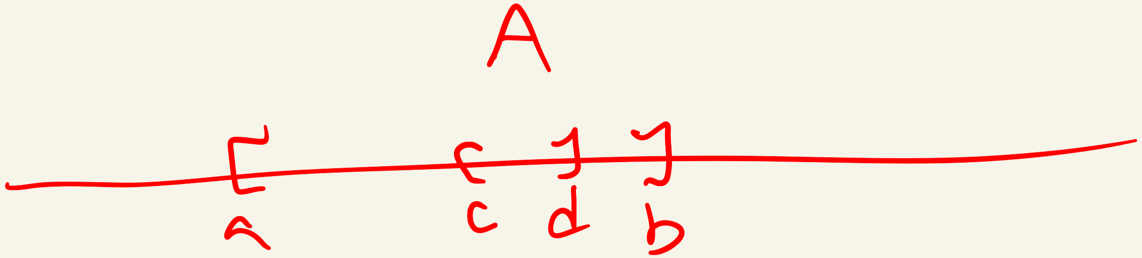
Joint

$$P(X \le k \cap Y \le l)$$

$$\int_{l_1}^{l_2}\int_{k_1}^{k_2} p(x,y)\,dx\,dy$$

$$p(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y} P(X \le k \cap Y \le l)$$

1D



$$p(x) = \frac{1}{b-a}$$

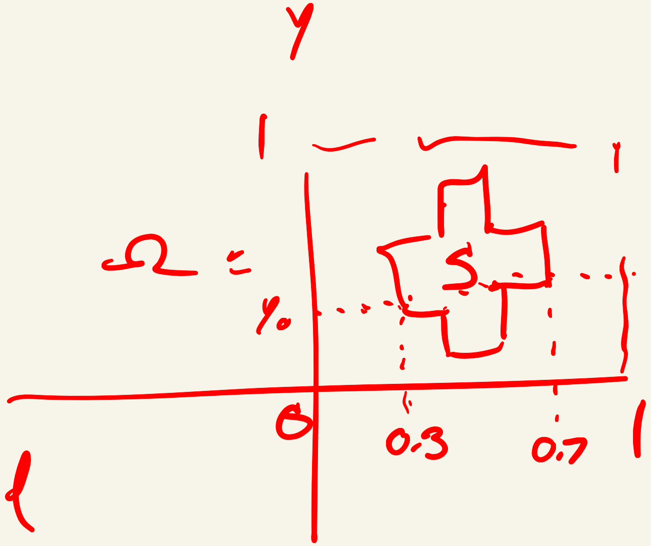
$$\mathbb{P}(A) = \int_c^d p(x)\,dx = \int_c^d \frac{1}{b-a}\,dx = \frac{x}{b-a}\Big|_c^d = \frac{d-c}{b-a}$$

$$\frac{\mathbb{P}(A)}{\mathbb{P}(\Omega)}$$

$$\frac{\text{Area}(A)}{\text{Area}(\Omega)}$$

2D

$\Omega =$



$$p(x=k) = \int_0^1 p(x=k, y=l)\,dl$$

$$p(x) = 1$$

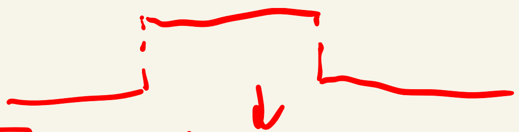
$$\int_0^1\int_0^1 1\,dx = 1$$

$$\mathbb{P}(S) = \frac{\text{Area}(S)}{\text{Area}(\Omega)}$$

$p(x,y) = \frac{1}{\text{Area}(S)}$ if $(x,y) \in S$; 0 otherwise

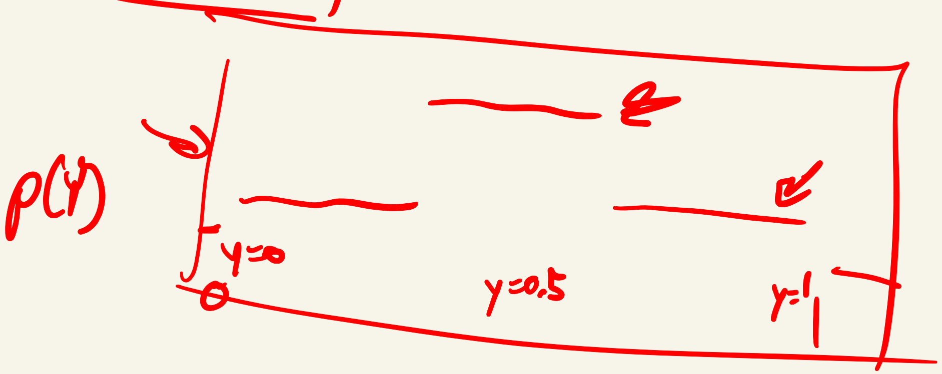
$Area(S) = 0.4$

$$p(x=x, y=k) = \begin{cases} 0 & \text{if } x < 0.3 \text{ or } x > 0.7 \\ \frac{1}{0.4} & \text{otherwise} \end{cases}$$



$$p(y=k) = \int_0^1 p(x=l, y=k)\, dl = (0.7-0.3)\frac{1}{0.4}$$

$= (\text{length of } S_i)(P)$



$$\mathbb{P}(A_1 \cup A_2 \cup \cdots \cup A_n) \le \sum \mathbb{P}(A_i)$$

Normal dist $\quad p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \quad \mathcal{N}(0,1)$

e^{-x^2}

1. Markov
2. Chebyshev
3. Union

Pro General

Con Weak

Pro Strong

Con Strict

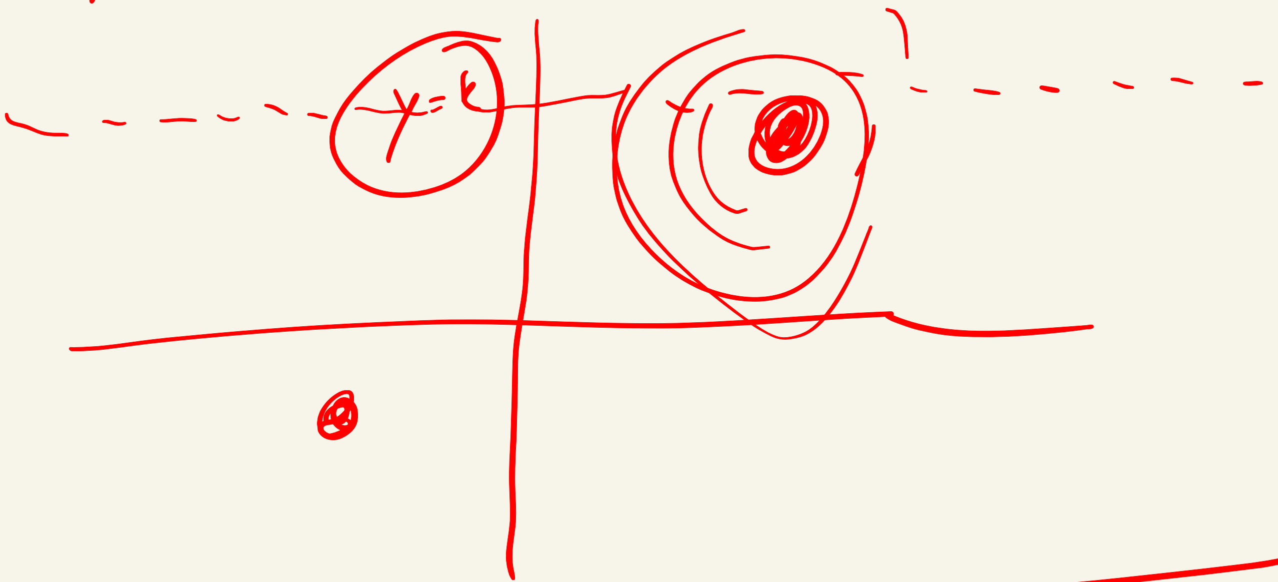
$\mathbb{P}(|X-\mu| \le \sigma)$

Pro General

Con Weak (for larger collections)

Cond Exp

$p(x,y)$



$$\mathbb{E}[X \mid Y=k] \text{ "="} \underset{\text{disc.}}{} \sum_{x \in \mathcal{X}} x\, \mathbb{P}(X=x \mid Y=k)$$

$$= \sum_{x \in \mathcal{X}} x \frac{\mathbb{P}(X=x, Y=k)}{\mathbb{P}(Y=k)} \quad (p_Y(k))$$

$$= \left(\frac{1}{\mathbb{P}(Y=k)}\right) \sum_{x=\mathcal{X}} x\, \mathbb{P}(X=x, Y=k)$$

cont.

$$= \frac{1}{p_Y(k)} \int_{-\infty}^{\infty} x\, p(x,k)$$

$\mathbb{E}[X \mid Y]$ is "takes value $\mathbb{E}[X \mid Y=k]$ w.p. $\mathbb{P}(Y=k)$ (or $p_Y(k)$)"

$\mathbb{E}[Y \mid X]$

$$\boxed{E[X] = E[E[X|Y]]} = \sum (\text{values})(\text{prob}_i)$$

"$$P(A) = \sum_{B_i} P(A|B_i)P(B_i)$$

$$= \sum_{k \in Y} E[X|Y=k]\,P(Y=k)$$

$$= \boxed{E[X]}$$

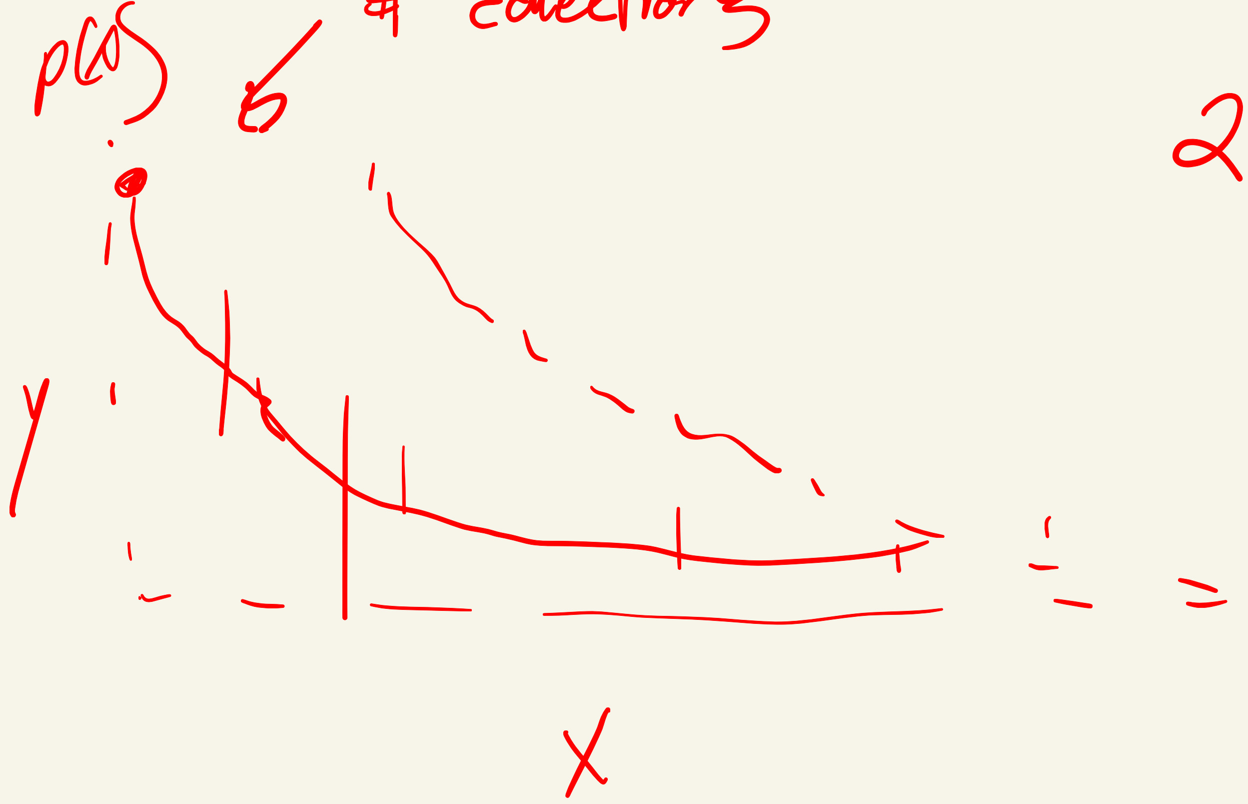
$$E[E[Y|X]] = E[Y]$$

$$E[X] = \sum x\,P(x)$$

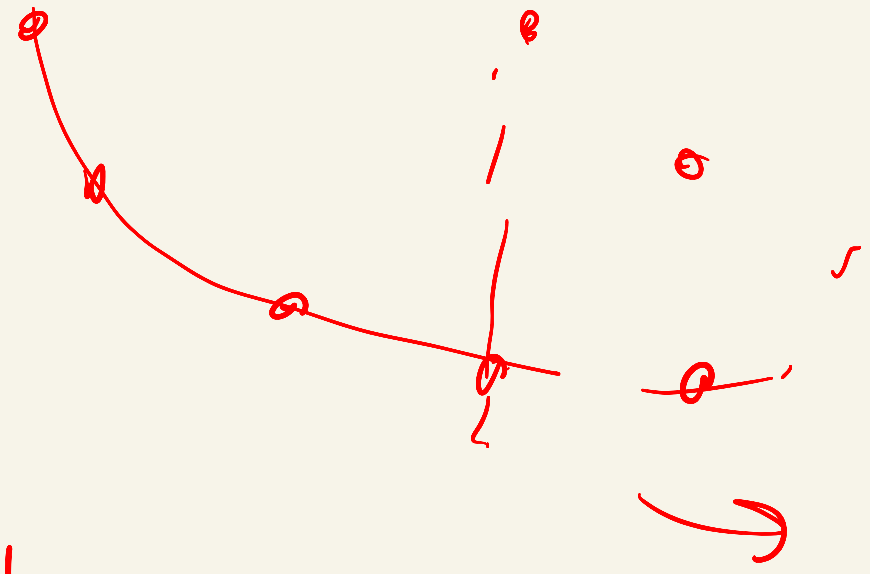
Geom $(1-p)^{k-1}p$

Exp | Half-life

" # collections



e^{-x}



$2^{-k} = \left(\frac{1}{2}\right)^k$

20

$$P(X \mid Y \geq 20) = \frac{P(X \cap X \geq 20)}{P(X \geq 20)}$$